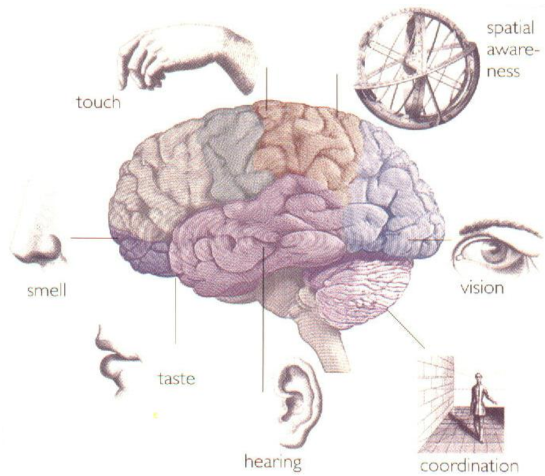
THE NEUROPSYCHOLOGY OF MATHEMATICS



Steven G. Feifer, D.Ed., NCSP, ABSNP
feifer@comcast.net

Presentation Of Goals

(1) Discuss the international trends in math, and reasons why the United States lags behind some industrialized nations in math and science.

(2) Introduce a *brain-based* educational model of math by identifying three basic neural codes which format numbers in the brain.

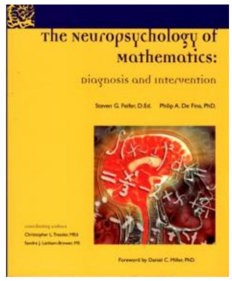
(3) Explore the role of various cognitive constructs including working memory, visual-spatial functioning, and executive functioning, with respect to math problem solving ability.

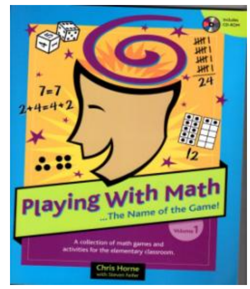
(4) Discuss three types of math disabilities, and specific remediation strategies for each type.

(5) Introduce the 90 minute assessment model of mathematics and interventions.

2

Future Reading Materials





www.schoolneuropsychpress.com

or

@schoolneuropsychpress

3

2011 TIMSS DATA

http://nces.ed.gov/pubs2013/2013009_1.pdf

Trends in International Mathematics and Science Study:
(5th Sample Collected Since 1995)

Grade 4: *TIMSS* assesses student knowledge in three content domains: number, geometric shapes and measures, and data display. **50%** of items consisted of number content

Grade 8: *TIMSS* assesses student knowledge in four content domains: number, algebra, geometry, and data and chance.

29% of items consisted of number content.

- **U.S. national sample consisted of 369 schools and 12,569 students at grade 4, and 501 schools and 10,477 students at grade 8. Data was collected from both public and private schools.**
- **In 2011, *TIMSS* was administered at grade 4 in 57 countries and at grade 8, in 56 countries.**

4

2011 TIMSS DATA: Grade 4

- The U.S. average mathematics score at grade 4 (541) was higher than the international TIMSS scale average, which is set at 500.3
- At grade 4, the United States was among the top 15 education systems in mathematics (8 education systems had higher averages and 6 were not measurably different) and scored higher, on average, than 42 education systems.
- Compared with 1995, the U.S. average mathematics score at grade 4 was 23 score points higher in 2011 (541 vs. 518).
- In 2011, the average science score of U.S. 4th-graders (544) was higher than the international TIMSS scale average, which is set at 500.
- At grade 4, the United States was among the top 10 education systems in science, and scored higher, on average, than 47 education systems.

5

2011 TIMSS DATA 4th Grade

Country	Average Score
International Average	*500*
1. Singapore	606
2. Korea	605
3. Hong Kong	602
4. Chinese Taipai	591
5. Japan	585
6. Northern Ireland	562
7. Belgium	549
8. Finland	545
9. England	542
10. Russian Federation	542
11. UNITED STATES	541
12. Netherlands	540
13. Denmark	537
14. Lithuania	534
15. Portugal	532
16. Germany	528
17. Ireland	527
18. Serbia	516
19. Australia	516
20. Hungary	515
21. Slovenia	513
22. Czech Republic	511
23. Austria	508

6

2011 TIMSS DATA: Grade 8

- The U.S. average mathematics score at grade 8 (509) was higher than the international TIMSS scale average, which is set at 500.
- At grade 8, the United States was among the top 24 education systems in mathematics (12 were not measurably different), and scored higher, on average, than 32 education systems.
- Compared with 1995, the U.S. average mathematics score at grade 8 was 17 score points higher in 2011 (509 vs. 492).
- In 2011, the average science score of U.S. 8th-graders (525) was higher than the TIMSS scale average, which is set at 500.
- At grade 8, the United States was among the top 23 education systems in science, and scored higher, on average, than 33 education systems.

7

2011 TIMSS DATA 8th Grade

Country	Average Score
International Average	*500*
1. Korea	613
2. Singapore	611
3. Chinese Taipai	609
4. Hong Kong	586
5. Japan	570
6. Russian Federation	539
7. Israel	516
8. Finland	514
9. *UNITED STATES	509
10. England	507
11. Hungary	505
12. Australia	505
13. Slovenia	505
14. Lithuania	502
15. Italy	498
16. New Zealand	488
17. Kazakhstan	487
18. Sweden	484
19. Ukraine	479
20. Norway	475
21. Armenia	467
22. Romania	458
23. United Arab Emirates	456
24. Turkey	452
25. Lebanon	449

8

PISA DATA (2009): 15 yr. olds
(Program for International Student Assessment)

A test of mathematical literacy for ***15*** year old students which focuses upon the direct application of mathematical principles. The test is administered every three years, with ***65*** countries participating in 2009. The test was ***not*** designed to measure curricular outcomes, but rather to assess mathematics' literacy within a real world context.

- **In 2009, the average U.S. score in mathematics literacy was 487, lower than the international average score of 496.**
- **Among the 65 countries in the sample, the U.S. was outperformed by 23 countries, and 12 countries had average scores not measurably different.**
- **There was no measureable change in the U.S. position when compared to the international average in both 2003 to 2006.**
- **U.S. boys scored 20 points higher than girls in math literacy (497 to 477)**

9

PISA DATA (2009): 15 yr. olds

Country	Average Score
International Average	496
1. Shanghai-China	**600**
2. Singapore	**562**
3. Hong Kong-China	**555**
4. Korea	**546**
5. Chinese Taipai	**543**
6. Finland	**541**
7. Liechtenstein	**536**
8. Canada	**527**
9. Netherlands	**526**
10. Macao-China	**525**
11. New Zealand	**519**
12. Belgium	**515**
13. Australia	**514**
14. Germany	**513**
15. Estonia	**512**
16. Iceland	**507**
17. Denmark	**503**
18. Slovania	**501**
19. Norway	**498**

10

PISA DATA (2009): 15 yr. olds

Country	Average Score
International Average	496
20. France	497
21. Slovak Republic	497
22. Austria	496
23. Polan d	495
24. Sweden	494
25. Czech Republic	493
26. United Kingdom	492
27. Hungary	490
28. Luxembourg	489
29. Ireland	487
30. Portugal	487
31. UNITED STATES	487
32. Spain	483
33. Italy	483
34. Latvia	482

11

The STEM Initiatives

- **Given the global demand for high tech workers, there is a greater exportation of jobs overseas due to a combination of cheaper wages, as well as a better educated workforce in mathematics and science.**
- **The US needs 400,000 new graduates in STEM fields by 2015. The STEM initiatives promote education in these fields and incorporate STEM into the curriculum.**
- **No Child Left Behind does not emphasize Science in the curriculum. Therefore, the applicational possibilities of mathematics often overlooked!**

12

4 Reasons for U.S Decline

1. The language of math matters! Building number connections centered around a base-10 principle is crucial in the development of mathematical efficiency when problem solving.
2. Dry and boring material. Mathematical skill building needs to be FUN, and therefore needs to be presented in the format of games and activities.
3. Too much focus on the answers. In order to become facilitators of mathematical knowledge, students should practice multiple methods of problem solving from both a visual-spatial and verbal approach.
4. Time on task. Most elementary math instruction occurs in the afternoon, just 45 minutes per day.

13

What is a Math Disability?

Basic Terminology:

- ***Math Disability (Dyscalculia)*- refers to children with markedly poor skills at deploying basic computational and cognitive processes used to solve equations (Haskell, 2000). These may include deficits with:**

  (1) Poor language and verbal retrieval skills
  (2) Working memory skills
  (3) Executive functioning skills
  (4) Faulty visual-spatial skills

*** There is no consensus definition of a true math learning disability at this time (Lewis, 2010).**
*** Approximately 6-14% of school age children have persistent difficulty with math (Mazzocco, Feigenson & Halberda, 2011)**

14

The "MLD" Profile
(Geary, 2011: Rasanen, et al., 2009)

1. Are slower in basic numeric processing tasks:
   - Rapidly identifying numbers.
   - Making comparisons between magnitude of numbers.
   - Counting forwards and backwards
2. Struggle in determining quantitative meaning of numbers:
   - Poor use of strategies.
   - Do not visualize numbers well.
3. Have difficulty learning basic calculation procedures needed to problem solve.

15

The "MLD" Profile
(Geary, 2011: Rasanen, et al., 2009)

MLD Error Profile:

- Prone to procedural errors such as saying "5,6,7" when solving 5 + 3 = ___
- Misalign numbers:
  36
  +3
  66
- Fail to borrow in a sequential manner:
  83
  - 44
  41
- Often deploy the wrong computational process:
  "The school store sold twice as many pencils to Sam than Robert. If Sam was sold four pencils, how many pencils were sold to Robert?" 8
- Poor retrieval of basic facts: 7 X 6 = 35

16

The Neural Machinery of Mathematics

Language Skills: (temporal lobes)

- Most Asian languages have linguistic counting systems past *ten* (*ten-one, ten-two, etc)* whereas English deviates from base-10 system (Campbell & Xue, 2001).
- In English counting system, decades come first then unit *(i.e. twenty-one)* or sometimes this is reversed *(i.e. fifteen, sixteen, etc…)*
- Chinese numbers are brief *(i.e. 4=si, 7=qi)* allowing for more efficient memory. By age four, Chinese students can count to 40, U.S. students to 15.
- U.S. kids spend 180 days in school
  South Korea children spend 220 days in school
  Japan kids spends 243 days in school

17

The Neural Machinery of Mathematics

Language Skills: (temporal lobes)

- Early math skills tend to be verbally encoded.
- Children with math disabilities frequently have delays in their language development. (Shalev et al, 2000)
- Word problems offer an intricate relationship between language and mathematics. Terms such as *all, some, neither, sum, etc.* may be confusing when embedded in the grammatical complexity of word problems (Levine & Reed, 1999).

18

The Neural Machinery of Mathematics

Working Memory Skills: (Baddeley,1998)

- ***Phonological Loop*** **- holds and manipulates acoustic information. Housed in *left temporal lobes*.**
- ***Visual-Spatial Sketchpad*** **- holds visual, spatial, and kinesthetic information in temporary storage by way of mental imagery. Housed along inferior portions of *right parietal lobes.***
- ***Central Executive System*** **- command post for controlling two slave systems. Allocates attention resources whereby two cognitive tasks can be executed. Primarily housed in *frontal lobes*.**
  - **Central executive system serves to inhibit any negative distractors when problem solving (Hopko, 1998).**

19

Working Memory In The Brain

Working Memory System	**Mathematical Skill**
• ***Phonological Loop***	• ***Retrieval of math facts***
	• ***Writing dictated numbers***
• ***Visual-Spatial Sketchpad***	• ***Mental math***
	• ***Magnitude comparisons***
	• ***Geometric Proofs***
• ***Central Executive System***	• **Inhibiting distracting thoughts**
	• **Modulating anxiety**
	• **Regulating emotional distress.**

20

Interventions for Lower Working Memory

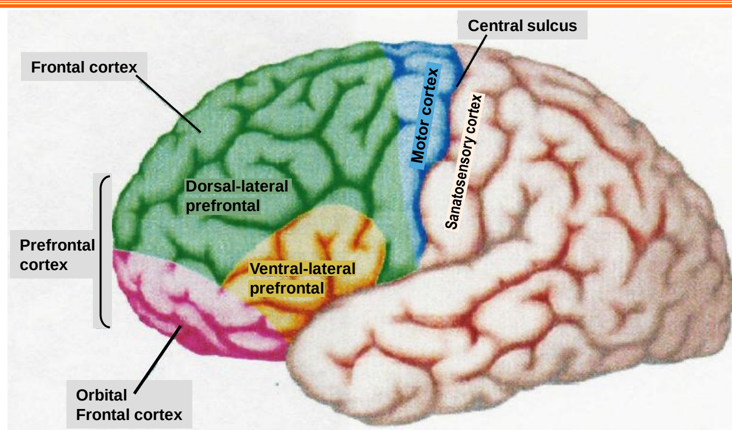
- **Number-line situated on student's desk.**
- **Use a calculator.**
- **Reduce anxiety in the classroom.**
- **Increase number sense through games such as dice, domino's, cards, etc..**
- **Encourage paper and pencil use while calculating equations.**
- **Use mnemonic techniques to teach math algorithm's and sequential steps to problem solving (i.e. The steps for long division are**
  - **<u>D</u>ivide, <u>M</u>ultiply, <u>S</u>ubtract, <u>B</u>ring <u>D</u>own:**
  - **<u>D</u>ad <u>M</u>om <u>S</u>ister <u>B</u>rother**
  - **<u>D</u>ead <u>M</u>onkies <u>S</u>mell <u>B</u>ad**

The Neural Machinery of Mathematics

<u>Executive Functioning Skills:</u> (frontal lobes)

- **Executive control mechanisms are a set of directive processes such as planning, self-monitoring, organizing, and allocating attention resources to effectively execute a goal directed task.**
- **Executive functioning dictates "*what to do when*", a critical process in solving word problems.**
- **Executive functioning allows students to choose an appropriate algorithm when problem solving.**

The Neural Machinery of Mathematics



- ***Orbital frontal cortex*** **is end point for ventral stream**
- ***Dorsal-lateral cortex*** **is end point for dorsal stream**

23

The Neural Machinery of Mathematics

EXECUTIVE DYSFUNCTION	BRAIN REGION	MATH SKILL
• *Selective Attention*	• *Anterior Cingulate/* • *Subcortical structures*	• Poor attention to math operational signs • Place value mis-aligned
• *Planning Skills*	• *Dorsal-lateral PFC*	• Poor estimation • Selection of math process impaired • Difficulty determining salient information in word problems

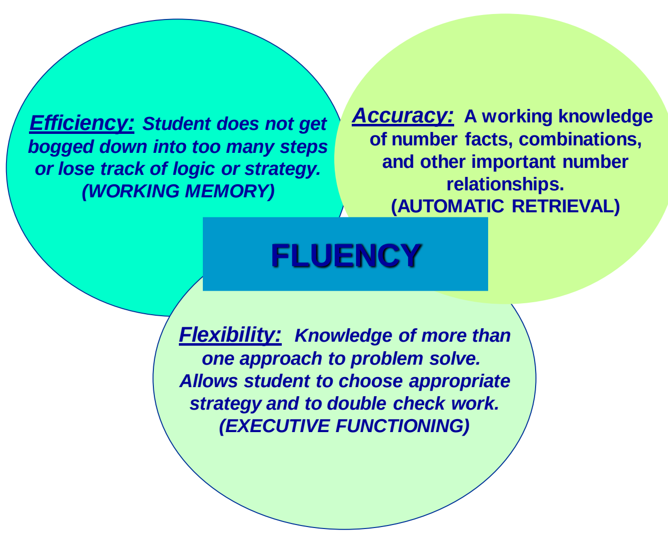
24

The Neural Machinery of Mathematics

EXECUTIVE DYSFUNCTION	BRAIN REGION	MATH SKILL
• *Organization Skills*	• *Dorsal-lateral PFC*	• Inconsistent lining up math equations • Frequent erasers • Difficulty setting up problems
• *Self-Monitoring*	• *Dorsal-lateral PFC*	• Limited double-checking of work • Unaware of plausibility to a response. • Inability to transcode operations such as (4X9) = (4X10) – 4

25

MATH FLUENCY (Russell, 1999)



26

Three Basic Neural Codes to Format Numbers in the Brain

(1) ***Verbal Code*** - numbers are encoded as sequences of words (*twenty-four* instead of 24).
- Dehaene & Cohen, 1997

- Left perisylvan region of temporal lobes.
- No need to understand quantitative concept.
- Main strategy used by younger children learning basic math facts (*two plus two equals four)*
- Critical for memorization of over-learned facts, such as multiplication facts (*nine times nine equals eighty-one).*

27

Three Basic Neural Codes to Format Numbers in the Brain

(2) Procedural Code - numbers are encoded as fixed symbols representing a quantity of some sort, and sequenced in a particular order. (*24 instead of twenty-four*). - Von Aster, 2000

I. Essential step to take numbers from a ***"word level"*** to a ***"quantitative level".***

- Circuitry involves the syntactical arrangement of numerals along our own internal number line. This requires an understanding of the five implicit rules of counting.

II. Critical in the execution of mathematical procedures for equations not committed to rote memory ***(i.e. subtraction with regrouping, long division, etc…).***

- Bi-lateral occipital-temporal lobes (DeHaene, 1997).

28

5 Rules of Counting
(Geary, 2004)

1) One-to-one correspondence – one verbal tag given to each object.
2) Stable order – word tags are unchanged (invariant) across counted sets.
3) Cardinality – the value of the final word tag represents the total quantity.
4) Abstraction – any object can be counted.
5) Order irrelevance – can count in any order.

* Rules of counting generally mastered by age 5.

* Math LD kids in 2nd grade have a poor conceptual understanding of counting rules, and adhere to *adjacency rule* (belief that you must count objects in a linear order)

29

Three Basic Neural Codes to Format Numbers in the Brain

(3) *Magnitude Code* - numbers are encoded as analog quantities. Allows for value judgements, such as *"9"* is bigger than *"4". (*Chocon, et al, 1999)

- Allows for semantic understanding of math concepts and procedures….."*Number Sense"*
- Allows for the evaluation of the plausibility of a response. (9 X 4 = 94)
- Allows for the transcoding of more challenging tasks into palatable forms of operations. For instance, 15 percent of 80 becomes 10 percent of 80 plus half the value.
- Bi-lateral horizontal inferior parietal lobes resides our approximate number system.

30

Approximate Number System
(Mazzocco, Feigenson & Halberda, 2011)

* A mental representational system of visual-spatial approximations that may underscore "number sense".
* Emerges independent of instruction (innate) and in non-humans as well.
* Distinguishes math LD from students from typical peers.
* Appears to be domain specific to math LD and may represent an inability to map number-words with visual spatial entities.
* Activation in inferior parietal sulcus.

http://www.nytimes.com/interactive/2008/09/15/science/20080915_NUMBER_SENSE_GRAPHIC.html

31

3 Subtypes of Math Disabilities

(1) ***Verbal Dyscalculia Subtype*:**
Main deficit is the automatic retrieval of number facts which have been stored in a linguistic code.

- Over-reliance on manipulatives when problem solving.
- Multiplication and addition often impaired.
- Poor at math fluency tests.
- Math algorithms often preserved.
- Often have learning disabilities in language arts as well.

KEY CONSTRUCT: Language & Verbal Retrieval

32

Verbal Dyscalculia Interventions
(Wright, Martland, & Stafford, 2000)

- Distinguish between reciting *number words,* and *counting* (map symbol to spatial value, not verbal tag).
- Develop a FNWS and BNWS to *ten, twenty*, and *thirty* without counting back. Helps develop an automatic retrieval skills (Al's Game, Chris' Game, Chip's Game)
- Develop a base-ten counting strategy whereby the child can perform addition and subtraction tasks involving tens and ones.
- Reinforce the language of math by re-teaching quantitative words such as *more, less, equal, sum, altogether, difference, etc…* (April's Game)

33

3 Subtypes of Math Disabilities

(2) *Procedural Dyscalculia Subtype*:
A breakdown in comprehending the syntax rules in sequencing numeric information.

- Difficulty recalling the algorithm or sequence of steps when performing longer math operations.
- Subtraction and division often impaired.

- Retrieval of math facts such as single digit addition, subtraction, and multiplication, as well as magnitude comparisons often preserved.
- Only partial development of "*number sense*"

Key Constructs: Working Memory and Anxiety

34

Procedural Dyscalculia Interventions

- Freedom from anxiety in class setting. Allow extra time for assignments and eliminate fluency drills.
- Mnemonic strategies(i.e. long division - Dead Monkies Smell Bad)
- Talk aloud all regrouping strategies.
- Use graph paper to line up equations.
- Adopt a curriculum such as *"Math Investigations"* which allows students to select their own algorithm.
- Attach number-line to desk and provide as many manipulatives as possible when problem solving.
- Teach skip-counting to learn multiplication facts.
- Teach patterns and relationships:
  (Melissa's Game, Mama's Game, Cordelia's Game, Habib's Game)

35

3 Subtypes of Math Disabilities

(3) ***Semantic Dyscalculia Subtype*:**
A breakdown in comprehending magnitude representations between numbers and understanding the spatial properties of numeric relations.

- Poor "number sense" and spatial attention.
- Difficulty evaluating the plausibility of a response (e.g. 2 X 4 = 24)
- Inability to transcode math operations into a more palatable form (e.g. 9 X 4 is same as (4 X 10) - 4).
- Poor magnitude comparisons.

Key Constructs: IQ, Executive Functioning, Visual-Spatial

36

Semantic Dyscalculia Interventions

- Teach students to think in ***"pictures"*** as well as ***"words".***
- Have students explain their strategies when problem solving to expand problem solving options.
- Teach estimation skills to allow for effective previewing of response.
- Have students write a math sentence from a verbal sentence.
- Construct incorrect answers to equations and have students discriminate correct vs. incorrect responses.
- Incorporate money and measurement strategies to add relevance. Use ***"baseball"*** examples as well.
- (Heidi's Game, Dwain's Game)

37

Evidenced Based Math Curriculums

Singapore Math – based upon math philosophy taught in Singapore…gained popularity after TIMSS study.

- Emphasis is on building upon math concepts so re-teaching is not needed, and little time devoted to reviewing previously taught skills before new concept taught.
- Flow of information is from Concrete to Pictorial to Abstract.
- The need for repetitive drill is minimized by logical sequencing of topics.
- The use of Bar-Models, which represent arithmetic quantities by line segments, facilitate understanding eliminate the need of rote memorization of facts.
- Word problems use to build semantic understanding of concepts.

38

Intervention Summary
(Feifer & Horne, 2007)

(1) Building number connections centered around a base-ten principle is crucial in the development of mathematical efficiency when problem solving.

(2) Mathematical skill building and developing a conceptual understanding of quantitative knowledge should be fun, self-motivating, and require far less effort when presented in the format of games and activities.

(3) In order to become facilitators of mathematical knowledge, students should practice multiple methods of problem solving by determining both a verbal and visual-spatial approach to solving addition, subtraction, multiplication, and division problems.

(4) Math instruction should promote student directed algorithms and not teacher directed ones.

39

The 90 Minute Mathematics' Assessment

- Intelligence Tests
- Visual-Spatial Functioning
- Working Memory Capacity
- Executive Functioning
- Attention Skills
- Math Skills and Number Sense
- Math Anxiety Scale
- Developmental and School History

40

Assessment Algorithm for Math: PAL II

- Oral Counting
- Fact Retrieval (Look & Write- Listen & Say)
- Computational Operations
- Place Value
- Part-Whole Relationships
- Finding the Bug
- Multi-Step Problem Solving
- Numeral Writing
- Numeral Coding
- Quantitative & Spatial Working Memory
- Rapid Automatic Naming
- Fingertip Writing

41

Assessment Summary for Math

1. Verbal Dyscalculia:
   - Poor counting
   - Poor rapid number counting
   - Slower retrieval of facts
   - Co-morbid reading/writing difficulties

2. Procedural Dyscalculia:
   - Difficulty lining up math equations
   - Forget math procedures
   - Better with single-digit facts than longer operations.
   - Co-morbid reading/writing difficulties

3. Semantic Dyscalculia:
   - Poor magnitude representation
   - Difficulty transcoding math operations
   - Poor estimation skills
   - Poor conceptual knowledge and quantitative thinking

42